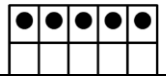
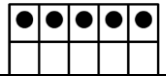
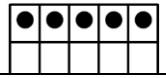
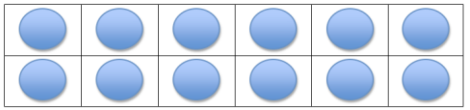
Maths Mastery

Progression in the use of manipulatives to support learning

USE IT!

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects	Real-life objects
0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards
Number track to 10	Number line to 20	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Numbered counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick
Tens frame 	Tens frame 	Tens frame 				
	Place value charts – Tens and ones	Place value charts – Hundreds, tens and ones	Place value charts – Thousands, hundreds, tens and ones	Place value charts – Ten thousands, thousands, hundreds, tens, ones and tenths	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Interlocking cubes - Use one colour to represent one amount	Interlocking cubes - Use one colour to represent one amount	Dienes	Dienes	Dienes	Dienes	Dienes
		Place value counters	Place value counters	Place value counters	Place value counters	Place value counters
	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – Th, H, T, O	Place value arrow cards	Place value arrow cards
Part-part-whole mat	Part-part-whole mat	Part-part-whole mat	Part-part-whole model	Part-part-whole model	Part-part-whole model	Part-part-whole model

Bar model with real-life objects	Bar model with real life objects/pictorial objects/representative objects eg. counters	Bar model with counters /Dienes progressing to numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers
Bead strings – ten	Bead strings - twenty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes
			Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount

Maths Working Wall – **DISPLAY IT!**		
Build it!	Use a real-life representation of the concept which children can see, touch and feel.	
Draw it!	Show a pictorial representation of the concept.	
Solve it!	Show the mathematical representation of the concept.	6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2 Factors of 12 are: 1, 2, 3, 4, 6 and 12
Practise it!	Encourage children to practice the concept. ***Interactive opportunity*** – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the concept.	1 x 2 = 2 2 x 2 = 4 3 x 2 = 6 etc.
Challenge it!	Set a challenge to be solved. ***Interactive opportunity*** – leave real-life objects or manipulatives for children to use to help solve the challenge.	How many different ways can 12 eggs be arranged into arrays? What if you try 24 eggs?
Say it!	Use vocabulary related to the concept	Multiply, times, repeated addition, array, divide, group, multiples, factors

Classroom Visual Prompts – SEE IT!

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Numicon number line with Numicon shapes	Numicon number line with Numicon shapes	Numicon number line	Fractions number line	Fractions and decimals number line	Fractions, decimals and percentages number line	Fractions, decimals and percentages number line
	Odd and even numbers	Odd and even numbers			Prime, square and cube numbers	Prime, square and cube numbers
	Number pairs totalling 10 Number pairs totalling 20	Number pairs totalling 10 Multiples of 10 totalling 100	Number pairs totalling 10 Multiples of 10 totalling 100			
0 – 10 number line / track	0 -20 number line	0 – 100 number line	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
	100 square	100 square	100 square	100 square		
Number names from 0 - 10	Number names from 0 - 20	Number names from 0 – 100	Number names from 0 - 1000	Number names to one million	Number names to one trillion	Number names to one trillion
Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins
	1, 2, 5 and 10 times tables	2, 3, 4, 5 and 10 times tables	All times tables up to 12 x 12	All times tables up to 12 x 12	All times tables up to 12 x 12	All times tables up to 12 x 12
			Roman numerals	Roman numerals	Roman numerals	Roman numerals
		< , > and = signs	< , > and = signs	<, > and = signs	<, > and = signs	<, > and = signs
Real-life / pictorial fractions	Real-life / pictorial fractions	Fractions including fraction number line/wall	Fractions including fraction number line/wall	Fractions including fraction number line/wall	Fractions, decimals and percentages including fraction number line/wall	Fractions, decimals and percentages including fraction number line/wall
						BODMAS
2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes

Progression in the teaching of counting in Foundation Stage

Pre-counting	Ordering	One to one correspondence	Cardinality (Knowing the final number counted is the total number of objects)
The key focus in pre-counting is an understanding of the concepts more, less and the same and an appreciation of how these are related. Children at this stage develop these concepts by comparison and no counting is involved.	Count by reciting the number names in order forwards and backwards from any starting point.	One number word has to be matched to each and every object. Lack of coordination is a source of potential error – it helps if children move the objects as they count, use large rhythmic movements, or clap as they count.	Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.
Pre-counting ideas *Provide children with opportunities to sort groups of objects explicitly using the language of **more** and **less.***   *Which group of apples has the most?* *Which group of apples has the least?*	**Ordering ideas** *Provide children with opportunities to count orally on a daily basis. Rote count so that children are able to understand number order and can hear the rhythm and pattern. Use a drum or clap to keep the beat.* 	**One to one correspondence ideas** *Play counting games together moving along a track, play games involving amounts such as knocking down skittles.* *Use traditional counting songs throughout the day ensuring children have the visual/kinaesthetic resources eg. 5 little ducks, 10 green bottles* 	**Cardinal counting ideas**  *How many bananas are in my fruit bowl? Allow children to physically handle the fruit.* *Provide children with objects to point to and move as they count and say the numbers.*

Progression in the teaching of counting in Foundation Stage

Subitising (recognise small numbers without counting them)

Children need to recognise small amounts without counting them eg. dot patterns on dice, dots on tens frames, dominoes and playing cards as well as small groups of randomly arranged shapes stuck on cards.

Abstraction

You can count anything – visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count things they cannot move (because the objects are fixed), touch (they are at a distance), see, that move around. Children also find it difficult to count a mix of different objects, or similar objects of very different sizes.

Conservation of number – MASTERY!

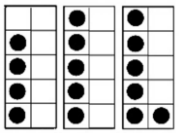
Ultimately children need to realise that when objects are rearranged the number of them stays the same.

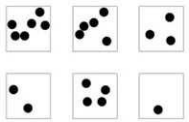
End of year counting expectations

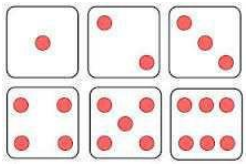
- count reliably to 20
- count reliably up to 10 everyday objects
- estimate a number of objects then check by counting
- use ordinal numbers in context eg first, second, third
- count in twos, fives and tens
- order numbers 1-20
- say 1 more/ 1 less than a given number to 20

Subitising ideas

Provide children with opportunities to count by recognising amounts.



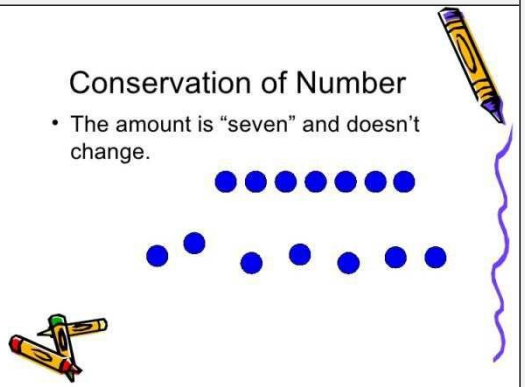




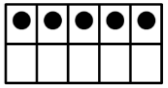
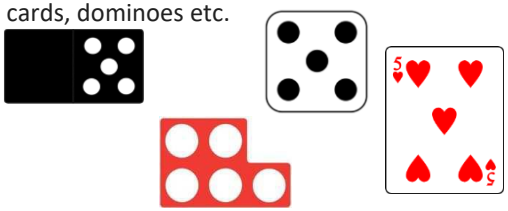
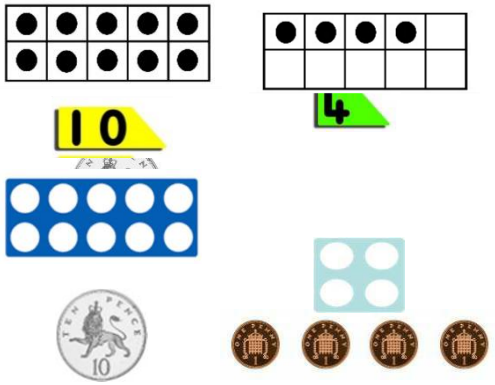
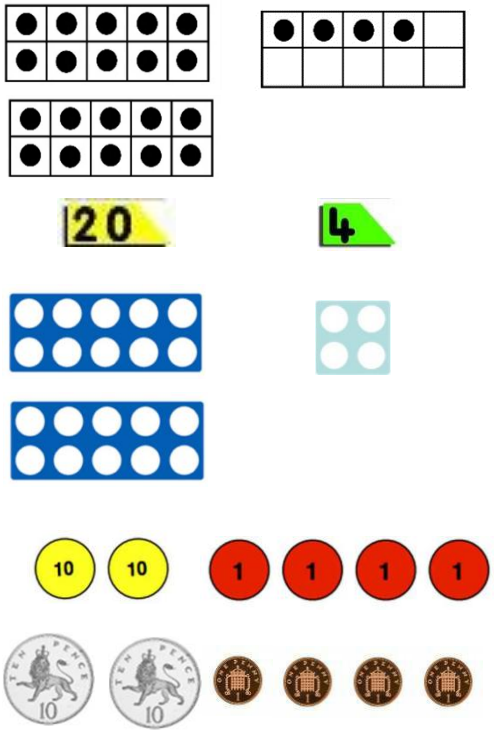
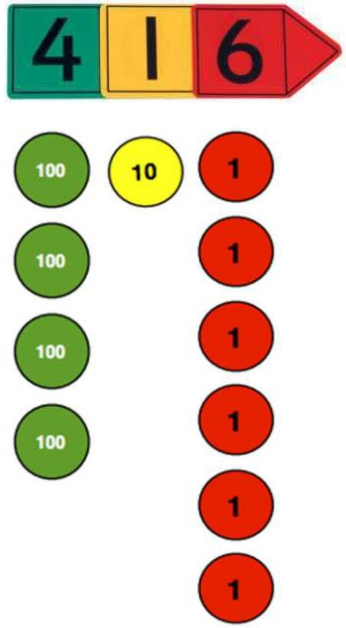
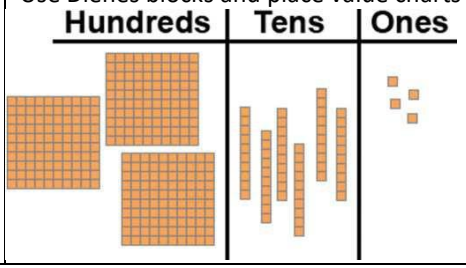
Abstraction ideas

How many pigs are in this picture?

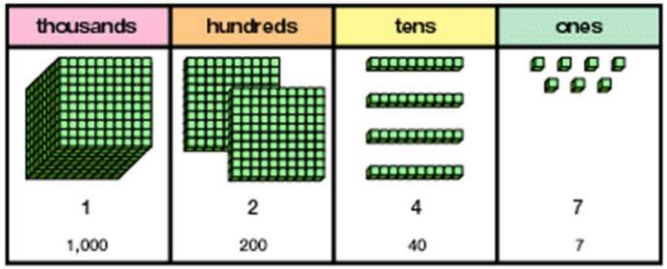
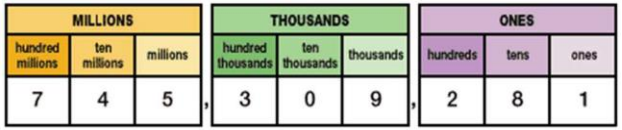
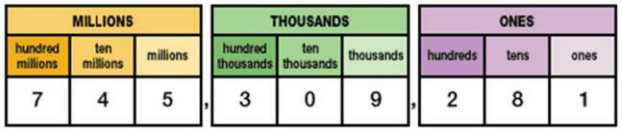
Provide children with a variety of objects to count.



Progression in the teaching of place value

Foundation	Year 1	Year 2	Year 3 onwards
Understanding ten	**Understanding numbers up to 20**	**Understanding numbers up to one hundred**	**Understanding numbers up to one thousand**
A TENS FRAME is a simple maths tool that helps children: • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value Use ***tens frames*** flash cards daily to ensure children recognise amounts. Use empty ***tens frames*** to fill with counters to enable children to understand number relationships. Either fill the ***tens frame*** in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.  Setting the counters in pairs, naturally allows the children to see addition concepts. Include other visual images such as dice, cards, dominoes etc. 	'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty-two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values. Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place-value understanding. 	Continue developing place value through the use of ***tens frames***. 	Continue developing place value through the use of manipulatives.  Use Dienes blocks and place value charts 

Progression in the teaching of place value

Year 4	Year 5	Year 6
Understanding numbers up to ten thousand	**Understanding numbers up to one million including decimals**	**Understanding numbers beyond one million including decimals**
Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters • Dienes blocks • Place value charts  	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimal counters) • Dienes blocks • Place value charts  	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimals counters) • Dienes blocks • Place value charts 

TENS FRAME IDEAS

LIFE SIZE TEN FRAME	Create a life-size ten frame in the classroom and outdoor play area. Use counters, pennies, teddies, gingerbread men, children etc.
FLASH	Flash ***ten frame*** briefly and have children write the number on a whiteboard. Using whiteboards, rather than having children say the number, ensures that all children attempt to respond and allows the teacher to assess class progress. When the response is oral, not all child responses are audible. Encourage children to share the different strategies used to find the total number of dots for cards, "How did you see it?" This can be varied by asking children to write the number and draw the pattern they saw, or by having them build the number flashed on their own blank frame.
FLASH: ONE MORE	Once children are familiar with the basic patterns, and know them automatically, flash a 10 frame or dot card and ask them to name the number that is one more than the number flashed. Variation: ask children to give the number that is two more/one less/double/ten more than the number flashed.
I WISH I HAD TEN	Flash a dot card or ten frame showing 9 or less and say, "I wish I had 10". Children respond with the part that is needed to make ten. The game can focus on a single whole, or the "wish I had" number can change each time. Variation: teacher flashes card and children write the complement of ten on individual whiteboards with dry erase markers.
I WISH I HAD 12	As above but children respond with how many more are needed to make twelve. Children should be confident in facts of 10 before this is attempted. For example to go from 8 to 12, they should realise they need 2 more to get to 10, then 2 more to 12. 2 and 2 is 4. Variation: Children draw an empty number line on their whiteboards to show the two jumps used to get to the target number.
1 MORE **1 LESS** **10 MORE** **10 LESS**	The following four prompts are written on the board: one more one less ten more ten less The teacher flashes a dot or ten frame card as the 'starting number'. The first child selects one prompt. For example, if the teacher flashes a card showing '5' the first child might say, "one more than 5 is 6", the second child might say, "ten more than 6 is 16", and the third child might say, "one less than 16 is 15". Continue until all children have had a turn.
TEEN FRAME FLASH (11-20)	**Teen Frame Flash (11-20)** Once children are subitizing ***ten frame*** patterns 0- 10, cards showing larger numbers (i.e. more than one ten frame) should be introduced. Use mental math sessions with the following key questions: How many? How many more than 10? As children become familiar with the 'teen' patterns introduce further questions to develop number relationships. • What is one more/two more than the number I flashed? • What is one less/two less than the number I flashed? • How far away is the number I flashed from twenty? • Double the number I flash. • What is the near Doubles fact? (i.e., if 15 is flashed, children answer 7+8)
MULTIPLES	Flash a ***tens frame*** and ask children to give you the product if the number you flash was multiplied by 2, 5 etc.

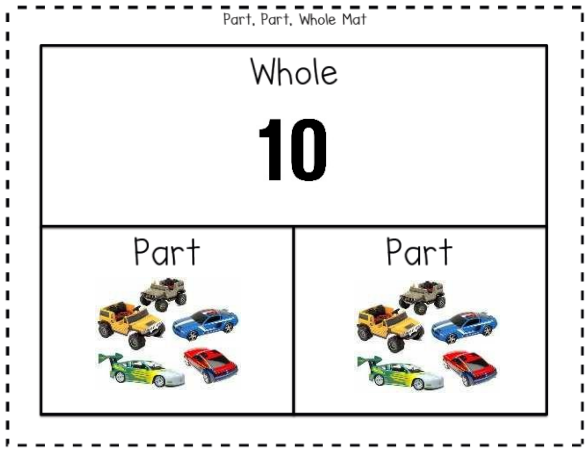
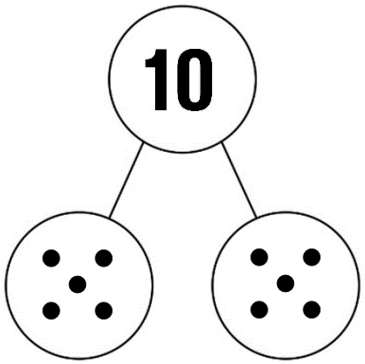
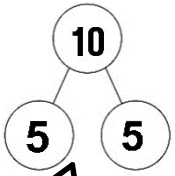
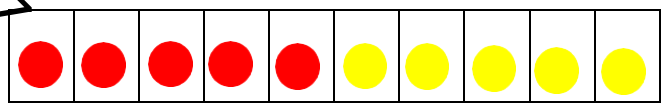
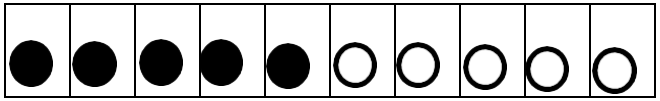
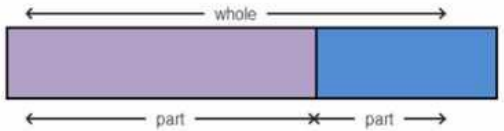
Progression in the teaching of calculations

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)

Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Progression in the teaching of calculations

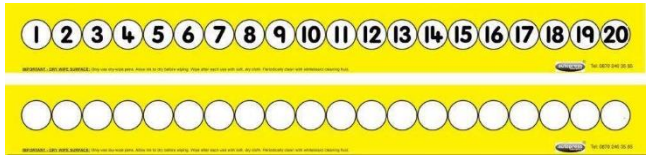
ADD IT!

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Combine two parts to make a whole model. Part-part-whole model			
Teach the children that the cubes/counters represent the real-life objects.	 	  Part + Part = Whole Whole – Part = Part	Use the part-part-whole diagram as shown above to move into the abstract 5 + 5 = 10 10 = 5 + 5
Use cubes to add two numbers together as a group or in a bar			Use the term ‘number sentence’.

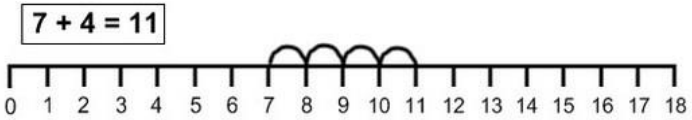
Start at the larger number and count on

Start with the larger number on the bead string then count on 1 by 1 to find the

Use counters on a number track to count on.



7 + 4 = 11



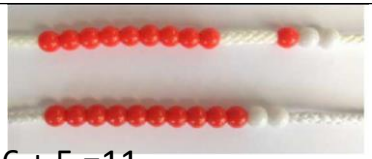
Start at the larger number on the number line and count on in ones or in one jump to find the answer.

4 + 7 = 11

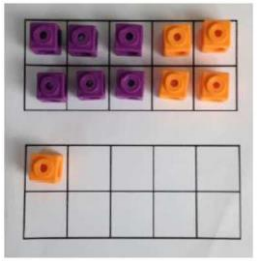
Place the larger number in your head and count on the smaller number to find your answer.

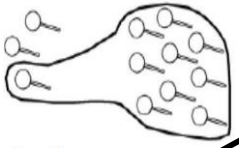
Regrouping to make 10.

Start with the bigger number and use the smaller number to make 10.



6 + 5 =11



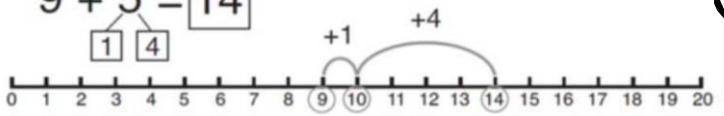


3 + 9 =

Use pictures or a number line. Regroup or partition the smaller number to make 10.

9 + 5 = 14

1 4



7 + 4= 11

If I am at seven, how many more do I need to make 10. How many more do I add on now?

Adding three single digits.

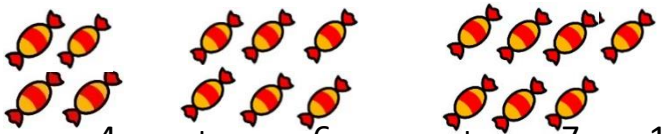
Encourage children to use known facts.

4 + 7 + 6= 17

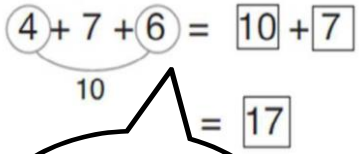
Put 4 and 6 together to make 10. Add on 7.

Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

Add together three groups of objects. Draw a picture to recombine the groups to make 10.

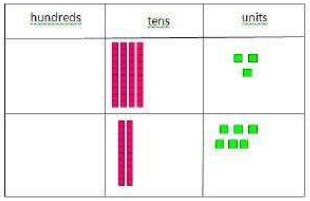


4 + 6 + 7 = 17



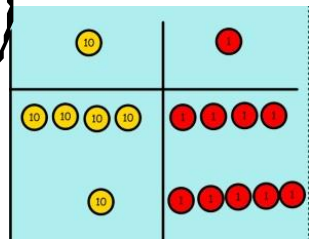
Combine the two numbers that make 10 and then add on the remainder.

Column method- no regrouping



$$\begin{array}{r} 43 \\ +\ 26 \\ \hline \end{array}$$

Use Dienes to add tens and ones before moving on to place value counters.



After practically using the base 10 blocks and place value counters, children can draw the Dienes to help them to solve addition calculations.

hundreds	tens	ones
	////	□ □ □
	//	□ □ □ □ □ □
	6	9

After practically using Dienes, children can draw the 'tens' and 'ones'.

Calculations

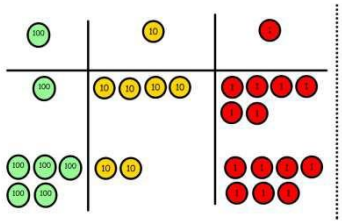
21 + 42 =

$$\begin{array}{r} 21 \\ +\ 42 \\ \hline \end{array}$$

Only select numbers which do not involve regrouping.

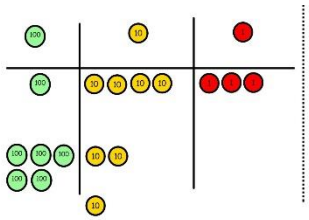
Column method- regrouping

Make both numbers on a place value grid.



$$\begin{array}{r} 146 \\ +\ 527 \\ \hline \end{array}$$

Add up the ones and exchange 10 ones for one 10 and so on



$$\begin{array}{r} 146 \\ +\ 527 \\ \hline \end{array}$$

This can also be done with Dienes to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.
Continue using place value counters as children begin to work with decimals

If necessary children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding

hundreds	tens	ones
/	////	□ □ □ □ □ □
/////	//	□ □ □ □ □ □ □
6	6	3
	1	

$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\ 1 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places to work with decimals

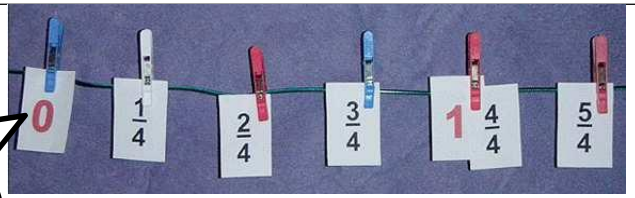
$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ 1\ 1 \end{array}$$

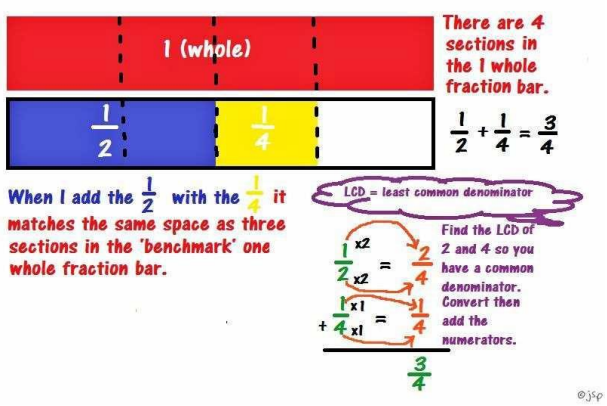
Then move onto decimals with a different number of decimal places

$$\begin{array}{rrrrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & \mathbf{0} \\ 5 & 9 & . & 7 & 7 & \mathbf{0} \\ + & 1 & . & 3 & \mathbf{0} & \mathbf{0} \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ 2 & 1 & & 2 & & \end{array}$$

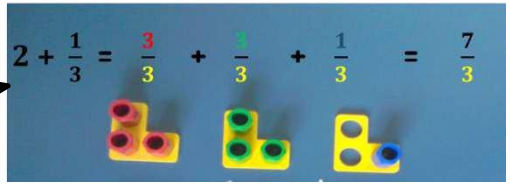
Add fractions

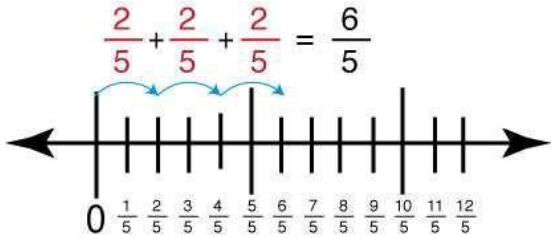
Count in fraction steps using real objects and a number line.



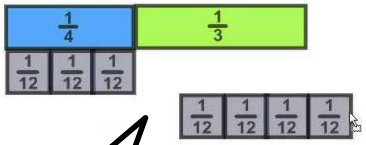


Use Numicon to add fractions.





$\frac{1}{4} + \frac{1}{3}$



Use the bar model to add fractions.

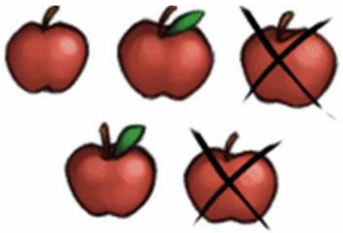
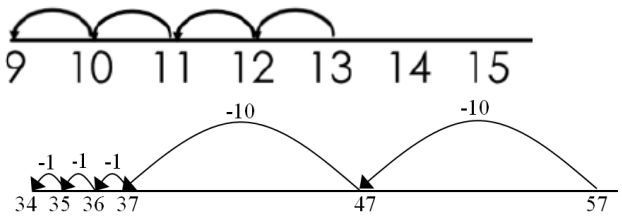
$\frac{1}{4} + \frac{1}{3} =$

$\frac{1 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$

$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

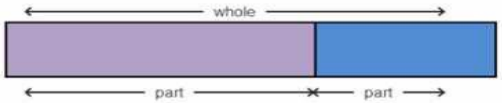
Progression in Calculations Policy

SUBTRACT IT!

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Taking away ones	Use real-life physical objects, counters, cubes etc. to show how objects can be taken away.  6 – 2 = 4   	Cross out drawn objects to show what has been taken away.  5 – 2 = 3	4 = 6 – 2 18 - 3= 15 8 – 2 = 6
Counting back Use counters and move them away from the group whilst counting backwards	Make the larger number in the subtraction calculation. Move the beads along the bead string whilst counting backwards in ones. 13 – 4  	Count back on a number line or number track Start at the bigger number and count back the smaller number showing the jumps on the number line. 	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Children will need regular practice counting backwards.

Use cubes to subtract a number from the bar.

Use the bar



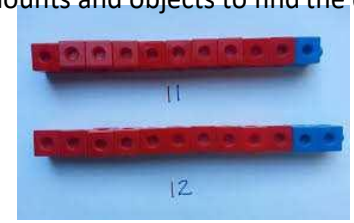
Part + Part = Whole

Whole – Part = Part

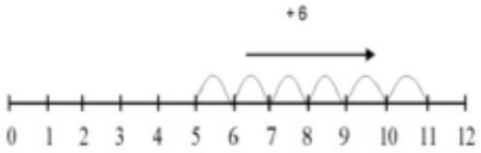
Find the difference

Use cubes to build towers or make bars to find the difference.

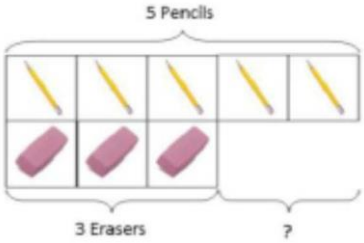
Compare amounts and objects to find the difference.



Count on to find the difference.

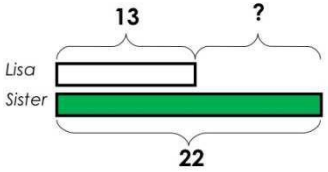


Use basic bar models with items to find the difference.



Comparison Bar Models

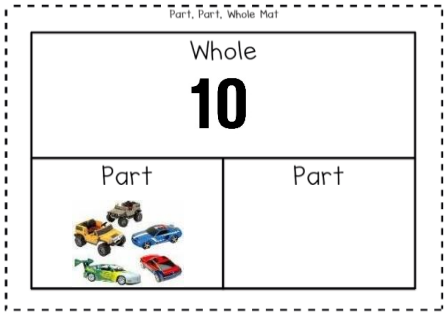
Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



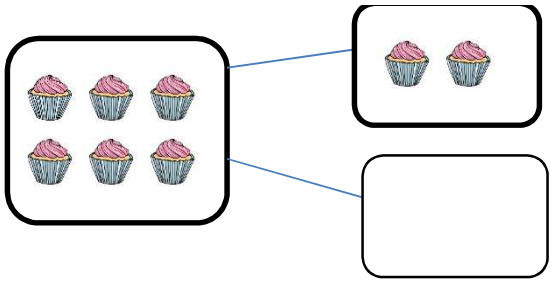
Draw bars to find the difference between two numbers.

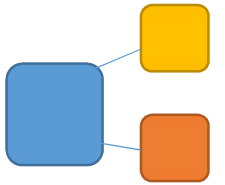
Hannah has 23 pencils, Helen has 15 pencils. Find the difference between the number of pencils.

Part-part-whole model



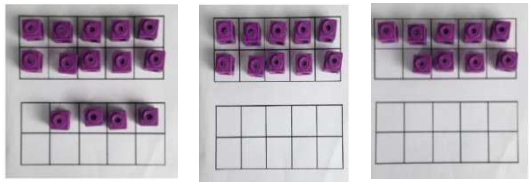
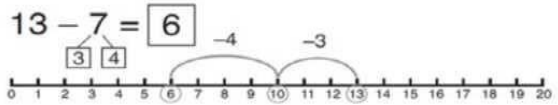
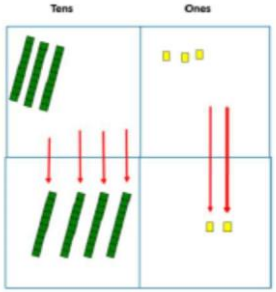
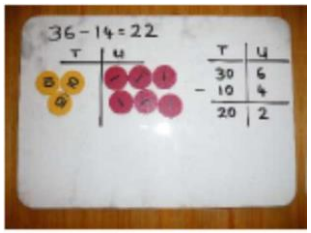
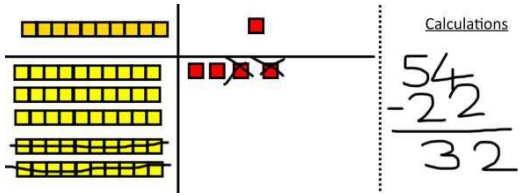
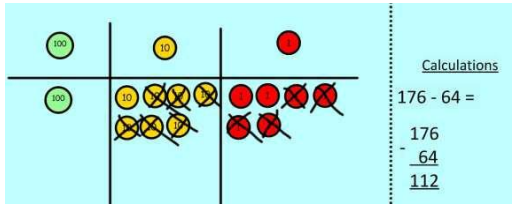
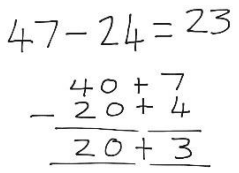
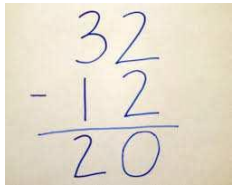
Use a pictorial representation of objects to show the part-part-whole model

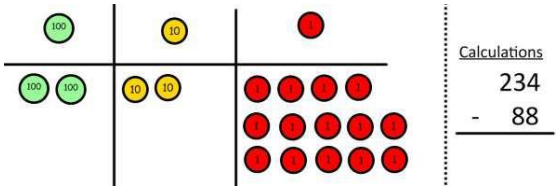
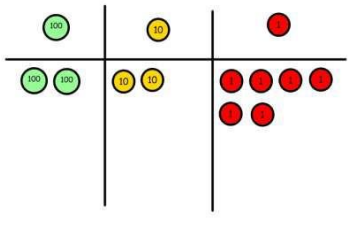
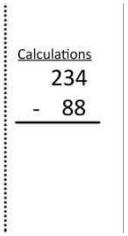
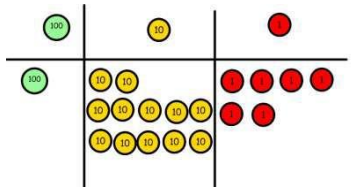
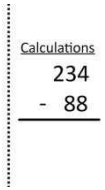
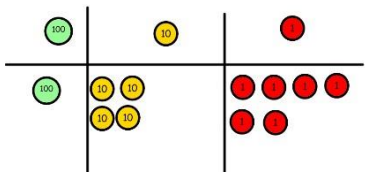
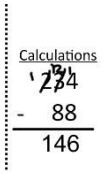
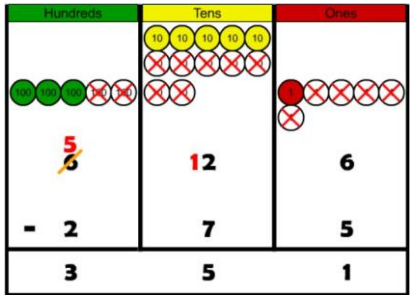
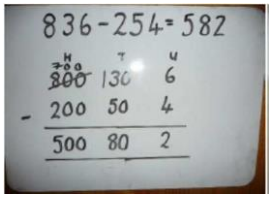
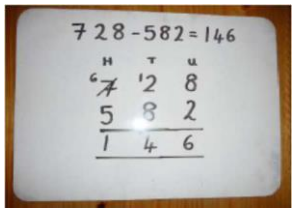
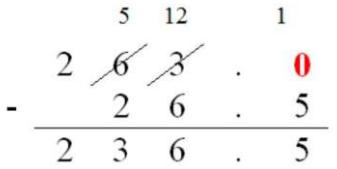




Link to addition – use the part whole model to help explain the inverse

Move to using numbers with the part-part-whole model

Make 10	14 – 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	75- 42 = Use Dienes to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first.  	Draw the Dienes or place value counters alongside the written calculation to help to show working.   	This will lead to a clear written column subtraction.  

Column method with regrouping	Make the larger number with the Dienes or place value counters. Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.   Calculations 234 - 88 Now I can take away eight tens and complete my subtraction   Calculations 234 - 88 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.   Calculations ~~23~~ 234 - 88 146	Draw the counters onto a place value grid and show what has been taken away by crossing the counters out as well as clearly showing the exchanges made. When confident, children can find their own way to record the exchange/regrouping.  	Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method.  This will lead to an understanding of subtracting any number including decimals. 

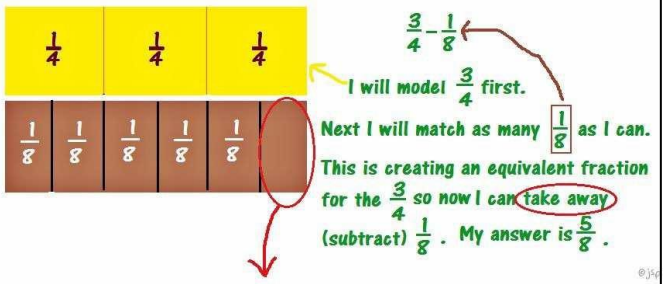
Subtract fractions

If there are five fifths and I eat one fifth, what fraction of the cake is left?

The cake has been divided into five slices. Each part is one fifth of the whole cake.

Draw a bar model to represent the cake.

Progress onto subtracting fractions with different denominators.



$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

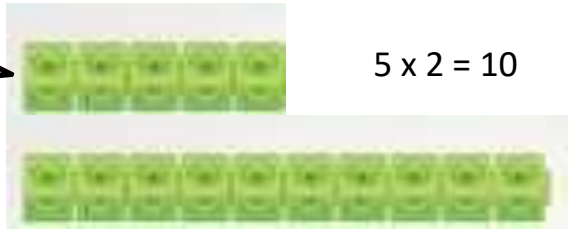
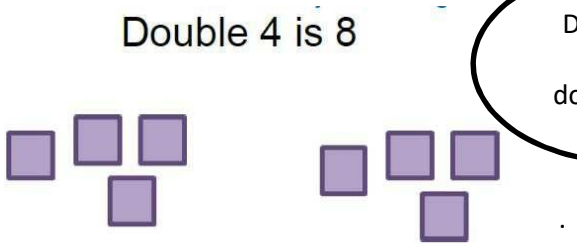
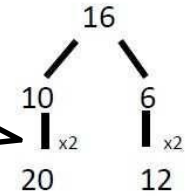
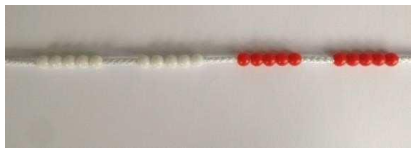
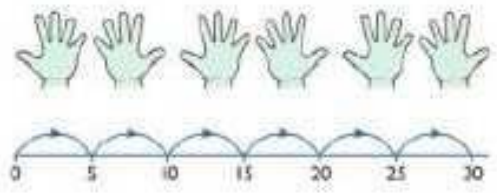
$\frac{3}{4} - \frac{1}{8} =$

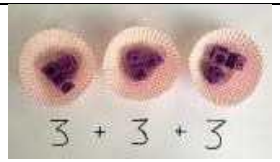
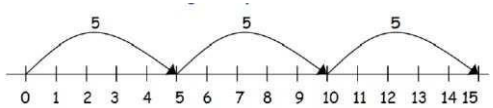
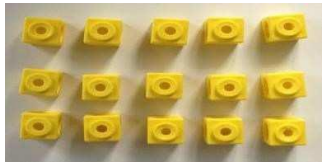
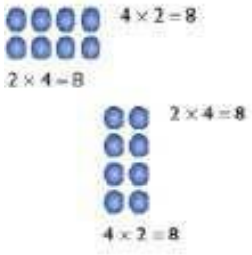
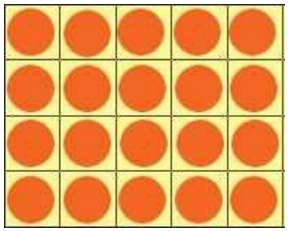
$\frac{3x2}{4x2} - \frac{1}{8}$

$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

Progression in Calculations Policy

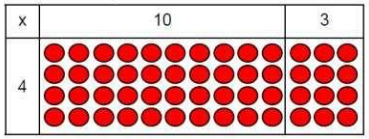
MULTIPLY IT!

Objective and strategies	Concrete DRAW IT!	Pictorial SOLVE IT!	Abstract BUILD IT/USE IT!
Doubling Double five is ten	Use practical activities to show how to double a number  5 x 2 = 10	Draw pictures to show how to double a number 	Double the 10 then double the 6  Partition a number and then double each part before recombining it back together
Counting in multiples	  Count in multiples supported by concrete objects in equal groups	 Use a number line or pictures to continue to support in counting in multiples	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2,4,6,8,10 5,10,15,20,25

Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

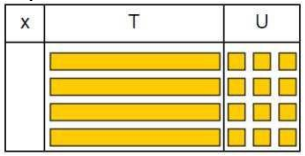
Grid Method

Show the link with arrays to first introduce the grid method.



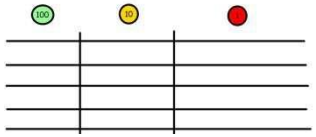
4 rows of 10
4 rows of 3

Use Dienes to move towards a more compact method.



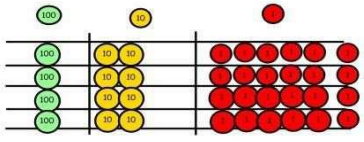
4 rows of 13

Use place value counters to show finding groups of a number eg. multiplying by 4 so we need 4 rows.



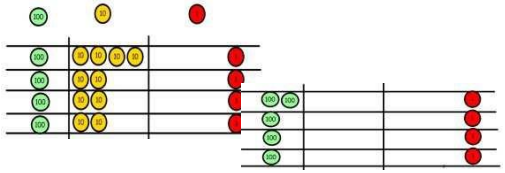
Calculations
4 x 126

Fill each row with 126.



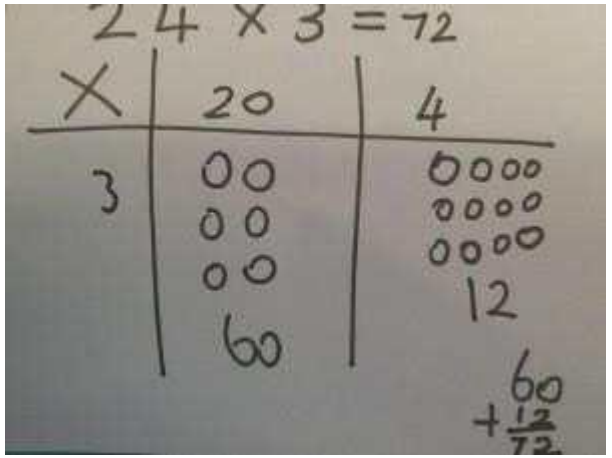
Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

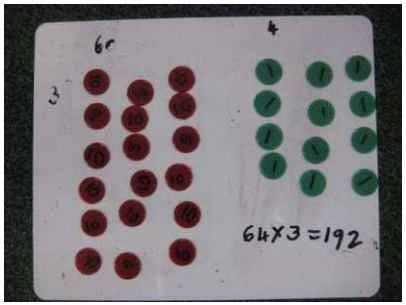
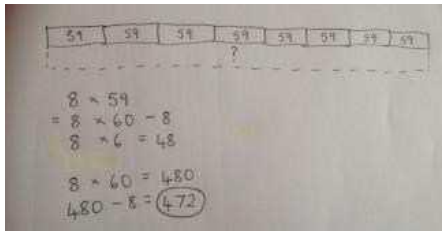
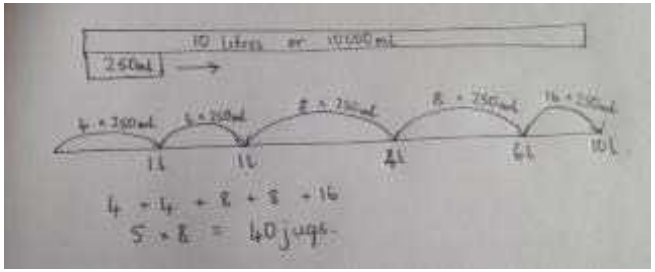
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

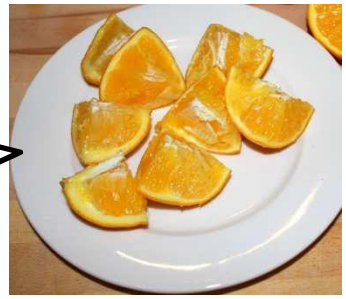
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

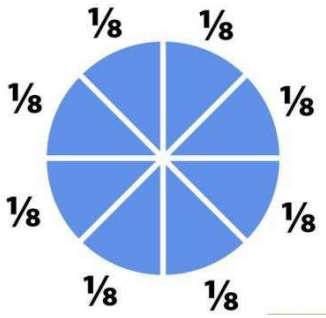
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 74 x 63 12 210 240 + 4200 4662 This moves to the more compact method. 327 x 53 981 ← 327 x 3 16350 ← 327 x 50 17331

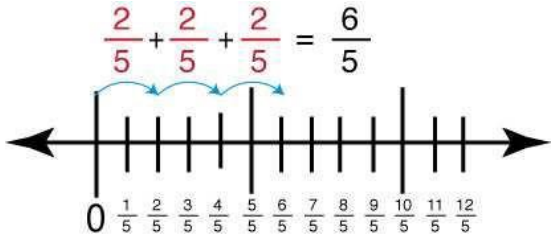
Multiplication of fractions

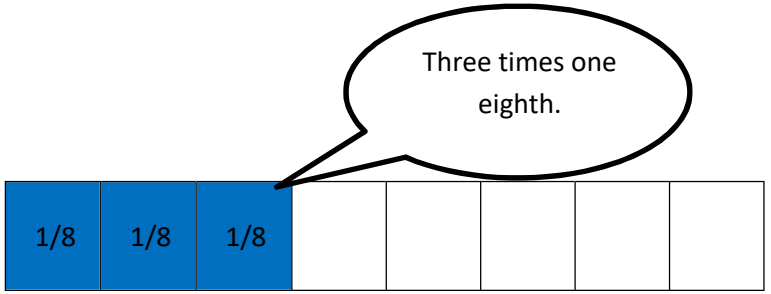
What would three lots of one eighth be?

Count in fraction steps (repeated addition)









$^{10}/_{8} = 1\ ^{2}/_{8}$
$^{9}/_{8} = 1\ ^{1}/_{8}$
$^{8}/_{8} = 1$
$^{7}/_{8}$
$^{6}/_{8}$
$^{5}/_{8}$
$^{4}/_{8} = {}^{1}/_{2}$
$^{3}/_{8}$
$^{2}/_{8} = {}^{1}/_{4}$
$^{1}/_{8}$

$3 \times {}^{1}/_{8} =$

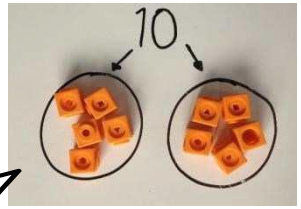
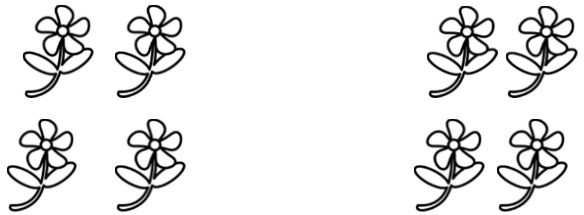
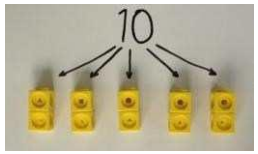
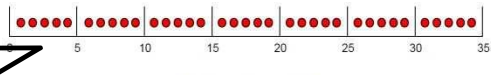
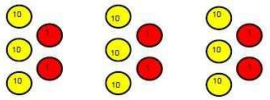
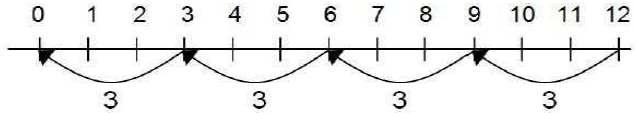
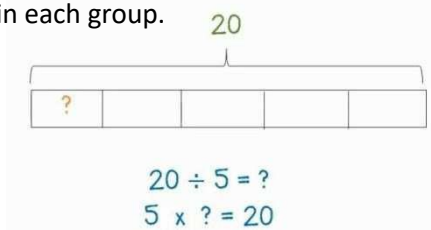
$\frac{3}{1} \times \frac{1}{8} = \frac{3}{8}$

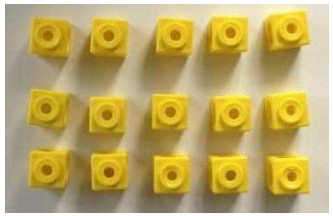
Multiply the numerators together then multiply the denominators

Progression in Calculations Policy

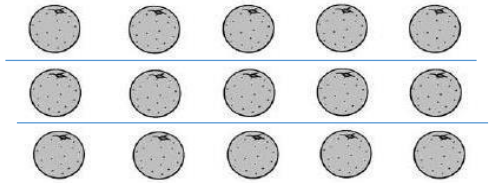
DIVIDE IT!

It is important to make links with fractions

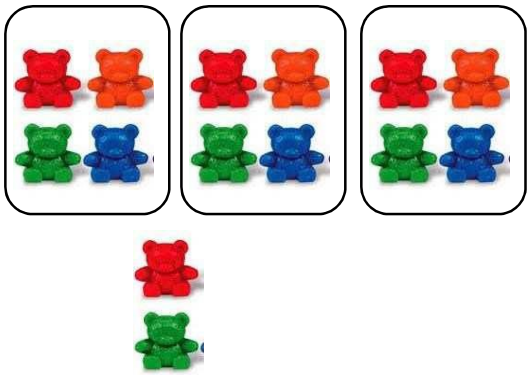
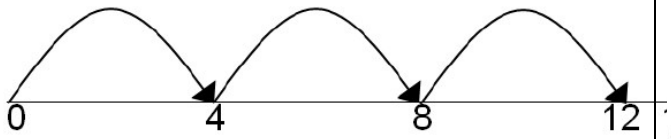
Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Sharing objects into groups If we are dividing by two we are finding one half.	 I have 10 cubes; can you share them equally into 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	One half of 14 is 7 ½ of 14 = 7 14 ÷ 2 = 7 Share 9 cakes between three people. 9 ÷ 3 = 3
Division as grouping If we are dividing by three we are finding one third	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

<table>
<tr>
<td>Division within arrays</td>
<td> 

Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15</td>
<td> 

Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

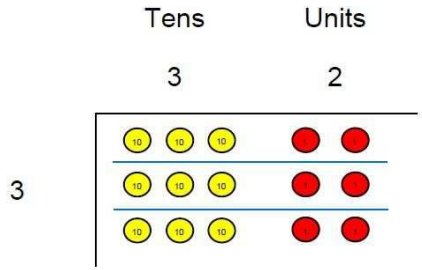
7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td>
</tr>
<tr>
<td>Division with a remainder</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over
  </td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.
 

Draw dots and group them to divide an amount and clearly show a remainder.
 
</td>
<td>Complete written divisions and show the remainder using r.
 
</td>
</tr>
</table>

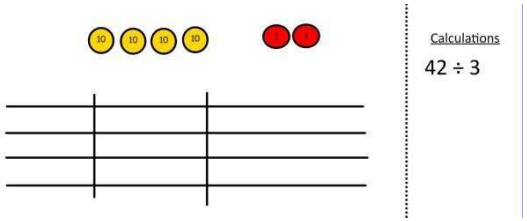
Short division

Find one third of 96.

96 ÷ 3 =

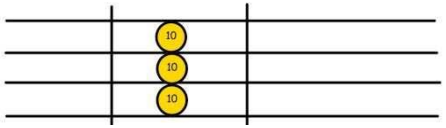


Use place value counters to divide using the bus stop method alongside

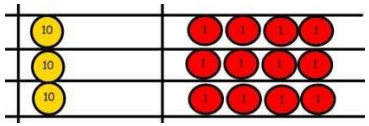


42 ÷ 3=

Start with the biggest place value; share 40 into three groups. Put 1 ten in each group then 1 ten left over.

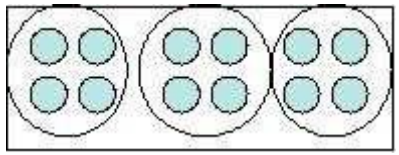


Exchange this ten for ten ones and then share the ones equally among the groups.



Look how much is in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4\overline{)872} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 86\text{ r }2 \\ 5\overline{)432} \end{array}$$

Long division (chunking method) Divide by single digit then progress to dividing by two digit number			86 r2 5 ⟌ 432 200 (40 x 5) 232 200 (40 x 5) 32 30 (6 x 5) 2 13 ⟌ 1937 - 1300 13 x 100 637 - 520 13 x 40 117 - 117 13 x 9 0
Division of fractions	½ ÷ 3 = Half of the pizza divided into 3 equal parts 	½ ÷ 3 =  Half of the bar divide into three equal parts	½ ÷ 3 = ½ ÷ 3/1 = ½ x 1/3 = 1/6

Times Table Policy

TIMES IT!

Times Tables are at the heart of mental arithmetic, which in itself helps form the basis of a child's understanding and ability when working with number. Once the children have learnt their times tables by heart, they are then able to work far more confidently and efficiently through a wide range of more advanced calculations

Reception	Year 1	Year 2	Year 3	Year 4	Year 5 and 6
I can count in steps of 1 I can count in steps of 2 I can count in steps of 10 I can count in steps of 5	I can count in steps of 5 I know my 1 times table I know my 2 times table I know my 10 times table	I know my 5 times table I know my 3 times table I know my 4 times table	I know my 6 times table I know my 7 times table I know my 11 times table	I know my 9 times table I know my 8 times table I know my 12 times table	Regular consolidation of all times tables

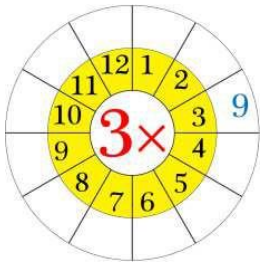
Rote learning

Times tables will be recited daily. Chant as: 'One times two is two, two times two is four, three times two is six …..'

Also chant as 'one multiplied by two is two, once two is two, one lot of two is two, one group of two is two, the product of one and two is two etc.'

1 x 1 = 1	2 x 1 = 2	3 x 1 = 3	4 x 1 = 4	5 x 1 = 5
1 x 2 = 2	2 x 2 = 4	3 x 2 = 6	4 x 2 = 8	5 x 2 = 10
1 x 3 = 3	2 x 3 = 6	3 x 3 = 9	4 x 3 = 12	5 x 3 = 15
1 x 4 = 4	2 x 4 = 8	3 x 4 = 12	4 x 4 = 16	5 x 4 = 20
1 x 5 = 5	2 x 5 = 10	3 x 5 = 15	4 x 5 = 20	5 x 5 = 25
1 x 6 = 6	2 x 6 = 12	3 x 6 = 18	4 x 6 = 24	5 x 6 = 30
1 x 7 = 7	2 x 7 = 14	3 x 7 = 21	4 x 7 = 28	5 x 7 = 35
1 x 8 = 8	2 x 8 = 16	3 x 8 = 24	4 x 8 = 32	5 x 8 = 40
1 x 9 = 9	2 x 9 = 18	3 x 9 = 27	4 x 9 = 36	5 x 9 = 45
1 x 10 = 10	2 x 10 = 20	3 x 10 = 30	4 x 10 = 40	5 x 10 = 50
1 x 11 = 11	2 x 11 = 22	3 x 11 = 33	4 x 11 = 44	5 x 11 = 55
1 x 12 = 12	2 x 12 = 24	3 x 12 = 36	4 x 12 = 48	5 x 12 = 60

Display



Times tables should be on display at the front of all classrooms, for children to use as support and reference.

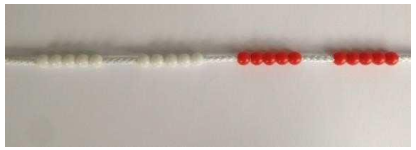
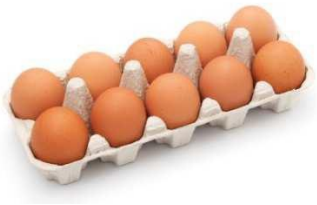
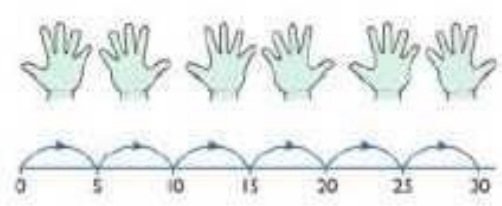
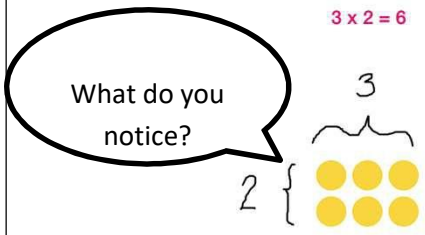
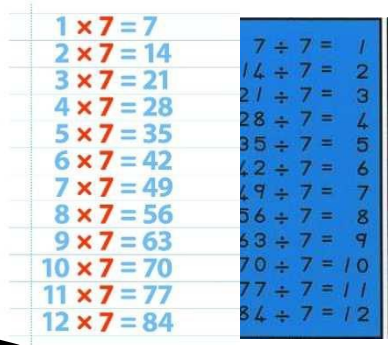
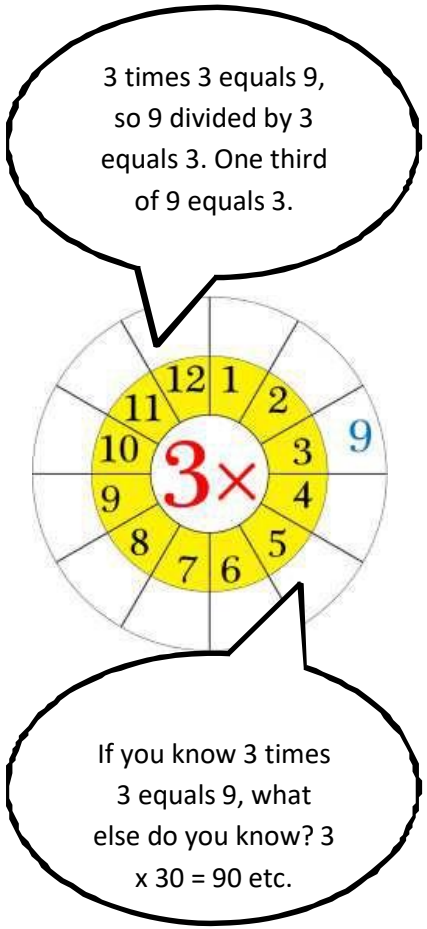
Year 1: 1, 2, 5 and 10 times tables should be displayed.

Year 2: 1, 2, 3, 4, 5 and 10 times tables should be displayed

KS2: All times tables up to 12 x 12 should be available for children. The display must be large enough for all children to see and on table top resources where necessary.

Individual times tables should be displayed.

Process of teaching times tables

Children will be taught the concept of multiplication using practical resources.	Children will progress on to using number lines or pictures.	Children will count in multiple steps.	Children will recite times tables by rote. Links will be made with 'grouping' and division whilst times tables are being taught.
Concrete **BUILD IT! / USE IT!**	**Pictorial** **DRAW IT!**	**Abstract stage 1** **SOLVE IT!**	**Abstract stage 2** **PRACTISE IT!**
Count in multiples supported by concrete objects in equal groups.   Use real-life arrays or build arrays.  	 Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30 Record multiplication number sentences.  Link multiplication and division facts.	Recite times tables by rote orally. 

COUNT IT!

Children need to rehearse counting regularly in order that they MASTER the number system.
Remember to count forwards and backwards orally and in written form.
Count from any number.
Ensure pronunciation of numbers is correct.

COUNTING IDEAS

Counting ladder – draw a ladder. Put starter number in the middle. Count forwards up the ladder and backwards down the ladder.	Chanting	Spot my error	Pass the parcel (wrap up numbers, predict next number)
Count in a sequence	Pendulum counting – multilink cube on a string	Speed counting	Mixed sequences eg +10, +1, -2 or missing number sequences
How many beats? Teacher beats wood block. Children count how many times in their head. Record. Each beat could represent an amount.	Action counting	Estimate and count When counting estimated objects, place the objects in rows of 10.	What am I counting in? Teacher counts, children work out rule. Can they then continue the pattern?
Counting stick (attached numbers then remove)	Count to the beat of the drum	Eyes closed counting game -blindfold one child, point to others who stand and say their name. Blindfolded child counts.	Play counting tennis eg count in steps, teacher says 5, children say 10 (mime using racket)
Fizz buzz	Use shapes eg triangles and count number of sides using 3 times table	Count coins in a pot, drop in one by one	Count using constant function on calculator

Lead the counting into calculation so the children see the link, for example, if counting in twos, calculate using repeated addition, multiplication – include inverse operations etc.

DIFFERENT WAYS OF COUNTING				
Single steps	Multiples	Use a rule eg 10 + 1 - 3	Missing numbers	Odds or evens
Fractions	Units of time	Millilitres/litres	Centimetres/metres	Decimals
Grams/kilograms	Negative numbers / Temperature	Percentages	Ordinals	Money

VISUAL AIDS FOR COUNTING				
Number line	100 square	Counting beads	Bead frame	Objects
Number snake	Number tiles	Pocket number line	Real money, large money or magnetic money	Shapes eg count sides
Counting stick	Whiteboards making own visual prompt	Objects (real life)	Base 10 Hundreds, tens, units	Groups of straws
Real life packaging showing arrays eg egg boxes, biscuit packets	Wrapping paper, wall paper etc. to count number of shapes	Number track	Counting bead string	Tape measure or metre stick
Clocks	Measuring jugs	Thermometer	Bead frame/abacus	Calculator
Pictures	Fingers	Interactive whiteboard	Multilink/buttons etc.	Number cards

REHEARSE IT!

Rehearsing old skills:
Children need to rehearse skills already taught to lead them to MASTERY.

The objectives will depend on your year group; however, it is important to keep old skills alive.

Remember to present the old skills in a variety of ways eg. Venn diagrams, Carroll diagrams, pictograms, tables, < and > signs, missing information, etc.

REASON IT!

There is a huge emphasis on reasoning in maths lessons. Children need opportunities to justify and explain their knowledge.
Ensure you are using:
NCETM reasoning questions
NCETM mastery documents
NRICH tasks

Odd one out	Would you rather have … ?	Find the mistake.	What is the same and what is different?
True or false?	Here is the answer, explain how it was worked out.	Always, sometimes, never	Give me a silly answer to this problem. What makes it silly?
Tell me about this…	Prove/disprove this statement.	Convince me that …	What if….?
Give me a hard and easy example of a calculation you could do with these numbers. Give me a hard and easy example of a five-digit calculation. Give me a hard and easy example of a question you could ask about this graph/pie chart etc.	What do you notice?	How are these linked?	If you know this fact, what else do you know? Eg. If you know: 4 + 6 = 10 You know: 40 + 60 = 100 100 – 40 = 60 The sum of 6 and 4 is 10. 4000 + 6000 = 10,000 100,000 – 60,000 = 40,000 If it is 6 o'clock now, in 4 hours it will be 10 o'clock.

RECALL IT!

Rapid recalling of key facts is important in developing fluency and MASTERY.
As children recall facts, deepen their knowledge by reasoning in context eg. When recalling number, bonds totalling 100: 'tell me two lengths that together make one metre.'

Recall number bonds	Recall addition / subtraction facts	Recall multiplication / division facts	Recall fraction, decimal, percentage equivalents
Recall shape names and properties	Recall time related facts	Recall measurement facts	

SAY IT!

Build mathematical vocabulary into every lesson.
Encourage children to speak in full sentences when giving responses.

Taboo – describe this word without saying it	How many words can you link to this word?	Match the word and its meaning.	Use a picture. How many mathematical words can you use?
Which of these words is the odd one out?	Write the definition of this word for someone who does not understand what it means.	Which word do these words link to?	Word of the day – use this word as many times in the day as possible (in context of course!)
Can you say a sentence which links these two words?	Tell me everything you can about this word.	Can you draw a picture to explain this word?	Hangman